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Wilfried Mirschel
Vitaly V. Terleev
Karl-Otto Wenkel *Editors*

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Forecasting Scanning Branches of the Hysteresis Soil Water-Retention Capacity for Calculation of Precise Irrigation Rates in Agricultural Landscapes Using a Mathematical Model

Vitaly V. Terleev, Wilfried Mirschel, Alex Topaj, Kirill Moiseev, Issa Togo, Yulia Volkova, Aleksandr O. Nikonorov, Roman Ginevsky and Viktor Lazarev

Abstract A mathematical model of the hysteretic soil water-retention capacity is proposed. Based on this model, a computer program called «Hysteresis» was developed. This program has options for identifying model parameters by the method of dot-fitting of experimental data, as well as for performing predictive calculations and

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V. V. Terleev (✉) · I. Togo · Y. Volkova · A. O. Nikonorov · R. Ginevsky · V. Lazarev
Peter the Great St. Petersburg Polytechnic University,
Polytechnicheskaya 29, St. Petersburg 195251, Russia
e-mail: vitaly_terleev@mail.ru

I. Togo
e-mail: issatogo@mail.ru

Y. Volkova
e-mail: yv1975@mail.ru

A. O. Nikonorov
e-mail: coolhabit@yandex.ru

R. Ginevsky
e-mail: rginevski@gmail.com

V. Lazarev
e-mail: lviktor.97@mail.ru

W. Mirschel
Leibniz-Centre for Agricultural Landscape Research (ZALF) e. V.,
Eberswalder Straße 84, 15374 Müncheberg, Germany
e-mail: wmirschel@zalf.de

A. Topaj · K. Moiseev
Agrophysical Research Institute, Grazhdanskii pr. 14, St. Petersburg 195220, Russia
e-mail: alex.topaj@gmail.com

K. Moiseev
e-mail: kir_moiseev@mail.ru

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graphical representation of the branches of the hysteresis loop. A series of computational experiments was performed in which the possibility of identifying the parameters of the mathematical model from the data on the main (boundary) branches of soil drying and wetting was investigated, and the accuracy of the predictive calculations of the scanning branches of the hysteresis loop was estimated. Data from the literature on four soils are used. The model has been compared with three models of predecessors. A sufficiently high accuracy of forecasting the scanning branches has been achieved. The practical value of the proposed model is the possibility of calculating precise rates for crop irrigation. Application of such rates: (i) prevents the percolation of excess moisture from the root layer of the soil; (ii) minimizes the loss of irrigation water, fertilizers, ameliorants and plant protection products and (iii) reduces the risk for groundwater contamination with agrochemicals and the threat of water eutrophication.

Keywords Water-retention capacity of soil · Hysteresis loop · Identification of parameters · Main (boundary) branches · Forecasting the scanning branches · Precise rates · Irrigation

17.1 Introduction

In irrigation farming, for every crop, it is necessary to take into account the physiological threshold characterizing the minimum permissible value of soil volumetric water content θ ($cm^3\ cm^{-3}$), at which yet negative irreversible changes in the growth and development of plants do not occur. Farther, for soil *humidity before irrigation*, we will use the designation HBI. In order to maximally moisten the soil, in irrigation practices it is usually applied the method to calculate the rates according to the difference FC-HBI, where FC is the *field capacity*, which characterizes the maximum possible supply of capillary-suspended water in the soil after the excess of gravitational moisture (percolation). The soil water-retention capacity (WRC) curve characterizes the change in the quasi-equilibrium states of water in form of a dependence of θ value on the capillary pressure of moisture ψ (cm H_2O). If the initial state of water corresponds to the complete saturation of the soil with moisture, then (taking into account the phenomenon of hysteresis) this curve describes the WRC main drying branch. On this branch the FC value corresponds to a certain capillary pressure ψ (FC). The capillary pressure averaged over the set of the WRC main drying branches for different soils is equal to -330 cm H_2O. On the main drying branch, the value $\psi = -330$ cm H_2O corresponds to the θ value for which we propose to use the term «*normative field capacity*» (NFC). In order not to confuse FC with NFC, for the measured field capacity of the soil we suggest using the refining term «*potential field capacity*» (PFC).

The determining factor of soil water-retention capacity is not the volume content of moisture in the soil, but the balance between capillary sorption forces and gravity. At the ψ (PFC) value, the capillary sorption forces counterbalance the force of gravity. With allowance for hysteresis, the change in quasi-equilibrium water states is characterized by two main branches and a multitude of scanning branches (drying and wetting) of the soil water-retention capacity. Any such scanning branch starts from the inversion point on the previous branch (main or scanning) and ends with another inversion point on the subsequent branch (main or scanning). The ψ (PFC) value corresponds to a set of θ values on different branches of the hysteresis loop. In this set, the maximum θ value is equal to the PFC. All other θ values at ψ (PFC) should be considered as values of the variable for which we propose to use the term «*effective field capacity*» (EFC).

Suppose that the PFC value is measured (for example, by the field flooding method). In the opinion of the authors, in this case the irrigation rate, calculated according to the PFC-HBI formula, is overestimated. This is explained as follows: if such rate is applied, immediately after watering the θ variable reaches the PFC value, but the corresponding value of the ψ variable on the wetting scanning branch is higher than ψ (PFC). At the same time, the excess of gravitational moisture, characterized by the difference in the PFC-EFC, flows off the root-inhabited soil layer. After the run-off of excess moisture, the ψ variable reaches the ψ (PFC), but the corresponding θ variable on the scanning drying branch turns out to be lower than PFC (θ = EFC < PFC). Incidentally, we note that if the NFC is higher than the PFC, then the irrigation rate calculated from the difference NFC-HBI is even more overestimated. Therefore, in order to prevent the unproductive loss of irrigation water, it is necessary to calculate the irrigation rate not according to the difference PFC-HBI (or NFC-HBI), but by the difference EFC-HBI. The EFC index is equal to the θ value at ψ (PFC) on the scanning wetting branch, which starts from the inversion point on the previous drying branch at ψ (HBI), and corresponds to the next watering (precipitation).

In order to estimate the EFC and predict the scanning hysteresis branches, it is proposed to use a physically adequate mathematical WRC model. It should be noted that in this method it is impossible to do without using such model, since it is impossible to measure an infinite set of scanning branches. The purpose of the study is to verify the WRC model underlying the proposed method for calculating the precise irrigation rates.

17.2 Theory and Mathematical Model

The property of the soil water-retention capacity is usually described in the form of functional dependence $\theta(\psi)$ (Brutsaert 1966; Ahuja and Swartzendruber 1972; Haverkamp et al. 1977). Now, there is no generally accepted physically adequate mathematical WRC model in the literature. Among the most famous models is the Van Genuchten model (Van Genuchten 1980). Along with the merits, Van Genuchten's model has a significant drawback, which lies in the fact that its parameters (α и n) have no physical meaning.

Following Kosugi (1994, 1996) and Hopmans (Kosugi and Hopmans 1998) on the basis of the concept of the lognormal pore size distribution (D'Hollander 1979) and the phenomenon of capillarity, the authors of this study described the functional dependence of the soil differential moisture capacity on ψ. An antiderivative function is obtained for the soil differential moisture capacity. This antiderivative, by definition, is a function of the soil integral moisture capacity, which describes the soil water-retention capacity in the form $\theta(\psi)$. Using the modified Winitzki approximation (2008), the continuous approximation for $\theta(\psi)$ function, which coincides with the Haverkamp et al. model (1977), was obtained, but which has an additional parameter (Terleev et al. 2015, 2016a, b). Thus the Haverkamp et al. (1977) model is physically justified and modified. Based on the physical meaning of the parameters of this modified model and using the assumption that the function of the soil differential moisture capacity assumes only two values corresponding to the sorption and desorption quasi-equilibrium moisture states, the authors of this study describe the hysteresis inherent in the soil water-retention capacity, using two sets of parameters: for the drying branches, the other for the wetting branches.

To describe the drying branch, starting from the point «*i*» on the wetting branch, the following formulas are used

$$\left\{\begin{array}{l} \left[\begin{array}{l} S_{e,d} = \left(1 + \left(\frac{\psi - \psi_{\text{ae}}}{\psi_{0,d} - \psi_{\text{ae}}}\right)^{n_d}\right)^{-1}, \\ \theta = \theta_R + \left(\theta_S^* - \theta_R\right) S_{e,d}, \\ \theta_s^* = \theta_s, \ \psi_{\text{we}} \le \psi_i, \ \psi < \psi_{\text{ae}}; \\ \theta_s^* = \theta_i, \ \psi_{ae} \le \psi_i < \psi_{\text{we}}, \ \psi < \psi_{ae}; \\ \theta_s^* = \frac{\theta_i - \theta_R\left(1 - S_{e,d}(\psi_i)\right)}{S_{e,d}(\psi_i)}, \ \psi_i < \psi_{\text{ae}}, \ \psi \le \psi_i; \end{array}\right. \\ \left[\begin{array}{l} \theta = \theta_s, \ \psi_{\text{we}} \le \psi_i, \ \psi_{\text{ae}} \le \psi \le \psi_i; \\ \theta = \theta_i, \ \psi_{ae} \le \psi_i < \psi_{we}, \ \psi_{\text{ae}} \le \psi \le \psi_i. \end{array}\right. \end{array}\right. \tag{17.1}$$

To describe the wetting branch, starting from the point «*j*» on the drying branch, the following formulas are used

$$\left\{\begin{array}{l} \left[\begin{array}{l} S_{e,w} = \left(1 + \left(\frac{\psi - \psi_{\text{we}}}{\psi_{0,w} - \psi_{\text{we}}}\right)^{n_w}\right)^{-1}, \\ \theta = \theta_R^* + \left(\theta_S - \theta_R^*\right) S_{e,w}, \\ \theta_R^* = \theta_j = \theta_r, \ \psi_j << \psi_{\text{ae}}, \ \psi_j \le \psi < \psi_{\text{we}}; \\ \theta_R^* = \frac{\theta_j - \theta_S S_{e,w}(\psi_j)}{1 - S_{e,w}(\psi_j)}, \ \psi_j < \psi_{\text{ae}}, \ \psi_j \le \psi < \psi_{\text{we}}; \end{array}\right. \\ \left[\begin{array}{l} \theta = \theta_S, \ \psi_j < \psi_{\text{ae}}, \ \psi_{\text{we}} \le \psi; \\ \theta = \theta_j = \theta_s, \ \psi_{\text{ae}} \le \psi_j, \ \psi_j \le \psi. \end{array}\right. \end{array}\right. \tag{17.2}$$

where $S_{e,d}$ and $S_{e,w}$ [dimensionless]—the effective moisture saturation of the soil for desorption and sorption quasi-equilibrium water states, respectively; θ_s (cm^3 cm^{-3})—the volumetric water content of the soil; θ_R (cm^3 cm^{-3})—minimum volumetric soil moisture at which water has the properties of a liquid; ψ_{ae} (cm H_2O) is the capillary pressure of the air entrance (bubbling pressure); ψ_{we} (cm H_2O) is the capillary pressure of the water entrance (pressure of displacement of the trapped air from dead-end pores); $\psi_{0,d}$ and $\psi_{0,w}$ (cm H_2O) are the values of capillary pressure at the most probable values of the random variable (the logarithm of the effective radius of the soil pore) for desorption and sorption quasi-equilibrium moisture states, respectively; σ_d and σ_{w} [dimensionless] are the values of the standard deviation of this random variable for desorption and sorption quasi-equilibrium moisture states, respectively, $n_d = 4\Big/\left(\sigma_d\sqrt{2\pi}\right)$ and $n_w = 4\Big/\left(\sigma_{\mathrm{w}}\sqrt{2\pi}\right)$ [dimensionless].

17.3 Results of Verification of the Mathematical Model

Based on the model described by formulas (17.1) and (17.2), the computer program «Hysteresis» was developed. For the verification of the model, literature data for four soils were used: *White silica sand* (Huang et al. 2005), *Dune sand* (Gillham et al. 1976), *Rideau clayey loam* and *Rubicon sandy loam* (Mualem 1976). The calibration of the model (identification of parameters) was carried out using the procedure of dot-fitting of experimental data on the main (boundary) branches of the hysteresis loop (Tables 17.1 and 17.2).

Using the parameterized model, predictive calculations of the scanning hysteresis branches were performed. The results were compared with the experimental data (Table 17.3). The proposed model was compared with three models of predecessors (Scott et al. 1983; Kool and Parker 1987; Huang et al. 2005) using the errors of the approximation of experimental data on the main (boundary) branches, as well as the errors of the prediction for the investigated hysteretic scanning branches. In Tables 17.2 and 17.3, the smallest errors in the calculations were given off in bold underlined font. From the analysis of Tables 17.2 and 17.3, it follows that the model proposed by the authors differs by the greatest number of results with the least error. For this model, with the parameter shown in Table 17.1, the artificial «pumping effect» was not detected.

Table 17.1 Parameters of the model (proposed by the authors) estimated from experimental data on the hysteretic main (boundary) branches using the dot-fitting procedure

Soils	Parameters							
	θ_R	θ_S	ψ_{ae}	ψ_{we}	$\psi_{0,d}$	$\psi_{0,w}$	n_d	n_{w}
White silica sand	0.0861	0.3574	−12.09	−1.797	−112.2	−41.42	3.996	2.287
Dune sand	0.0934	0.3010	−19.82	−3.594	−33.68	−19.99	3.170	3.298
Rideau clayey loam	0.2896	0.4179	−20.00	6.26	−66.96	−29.44	1.951	1.999
Rubicon sandy loam	0.1688	0.3829	−13.00	16.00	−88.42	−36.32	2.911	2.993

Table 17.2 The mean absolute difference between the results of calculating the hysteretic main (boundary) branches (by the dot-fitting procedure) and the experimental data

Soils	Comparable models			
	Scott et al. (1983)	Kool and Parker (1987)	Huang et al. (2005)	Proposed
White silica sand	0.0028	0.0107	0.0030	**0.0019**
Dune sand	0.0027	0.0080	0.0031	**0.0023**
Rideau clayey loam	**0.0032**	0.0057	0.0057	**0.0032**
Rubicon sandy loam	**0.0045**	0.0130	0.0055	0.0098

On Figs. 17.1a, b, 17.2a, b, 17.3a, b and 17.4a, b the measured data are shown by points; the results of the approximation (dot-fitting) for the main (boundary) branches, as well as the results of the predictive estimation for the scanning branches of hysteresis loop (using the model proposed here) are shown by solid curves. Based on the results of the computational experiments, a comparative analysis for an accuracy of the predictive estimating the hysteretic scanning branches was carried out. The model proposed here and three models of predecessors were used (Scott et al. 1983; Kool and Parker 1987; Huang et al. 2005).

17.4 Discussion

The idea of using the Haverkamp et al. model (1977) to describe hysteresis is not new. An example is the model of the hysteresis of the soil water-retention capacity, proposed in (Scott et al. 1983), where an original solution was found for scanning branches starting from turning points. Nevertheless, the hysteresis models based on the dependencies proposed by Haverkamp et al. (1977) and Van Genuchten (1980) in the literature (Huang et al. 2005) have been criticized for the potential manifestation of the «pumping effect» . This criticism is true, but—in part. The «pumping effect» is that when the values of the capillary pressure of soil moisture oscillate in a fixed interval, the values of volumetric water content «drift» , and it is possible an intersection between the scanning and main branches, that is accompanied by «exit» of the scanning branches out of the physically acceptable area. Indeed, the dependencies $\theta(\psi)$ proposed in (Haverkamp et al. 1977; Van Genuchten 1980) contain the potential manifestation of this undesirable artificial effect in the models of hysteretic soil water-retention capacity. However, this effect does not always appear, but only with certain combinations of parameters. If the values of the parameters do not exceed the limits of the physically acceptable area, then the «pumping effect» does not appear. Of course, such boundaries can only be defined for physically adequate models. The model proposed by the authors belongs to this class of models.

Table 17.3 The mean absolute difference between the results of calculating the hysteretic scanning branches (using four models) and the experimental data

Soils	Scanning branches	Comparable models							
		Scott et al. (1983)		Kool and Parker (1987)		Huang et al. (2005)		Proposed	
		Wetting	Drying	Wetting	Drying	Wetting	Drying	Wetting	Drying
White silica sand	Primary	0.0033	0.0070	0.0035	**0.0028**	0.0035	0.0066	**0.0024**	0.0087
	Secondary	0.0029	0.0035	0.0054	0.0095	0.0050	0.0031	**0.0014**	**0.0028**
	Tertiary	0.0099	0.0128	0.0130	0.0137	**0.0082**	**0.0042**	0.0130	0.0149
Dune sand	Primary	0.0074	0.0096	0.0096	0.0151	**0.0057**	0.0096	0.0067	**0.0095**
Rideau clayey loam	Secondary	0.0038	**0.0050**	**0.0024**	0.0065	0.0034	0.0071	**0.0024**	0.0062
Rubicon sandy loam	Tertiary	0.0106	0.0141	0.0118	0.0175	0.0076	**0.0105**	**0.0052**	0.0108

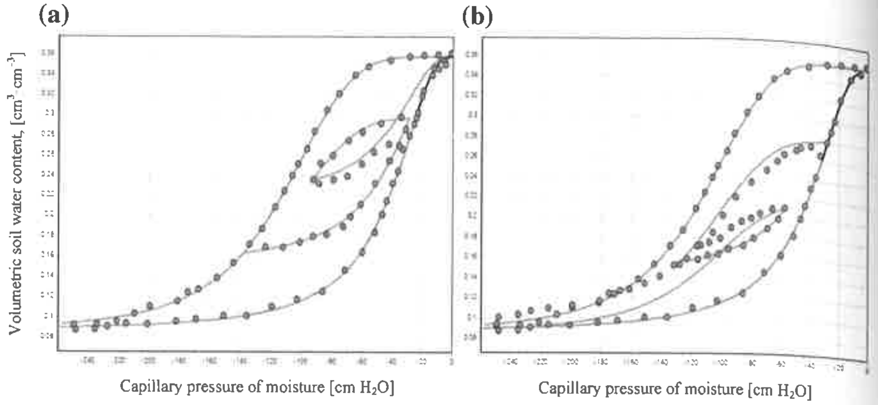

Fig. 17.1 **a** Approximation (dot-fitting) of measured data about the main branches; predictive estimation of the wetting primary branch, the drying secondary branch, the wetting tertiary branch for soil *White silica sand* using the proposed model. **b** Approximation (dot-fitting) of measured data about the main branches; predictive estimation of the drying primary branch, the wetting secondary branch, the drying tertiary branch for soil *White silica sand* using the proposed model

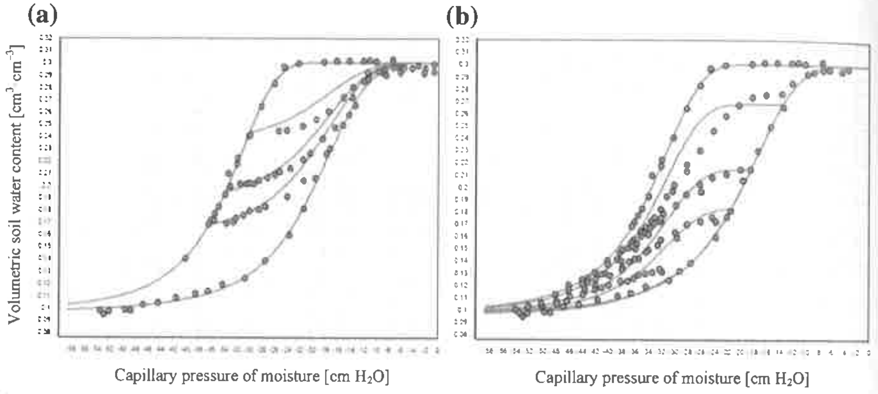

Fig. 17.2 **a** Approximation (dot-fitting) of measured data about the main branches; predictive estimation of the three wetting scanning branches for soil *Dune sand* using the proposed model. **b** Approximation (dot-fitting) of measured data about the main branches; predictive estimation of the four drying scanning branches for soil *Dune sand* using the proposed model

Thus, the obvious drawback of any model of the hysteretic soil water-retention capacity based on the functional dependence $\theta(\psi)$ proposed by Van Genuchten (1980) is the fundamental impossibility to determine for the parameters the boundaries of the physically feasible area (since the parameters α and n of the given function (1980) have no physical meaning): hence, in any such model an emergence of the «pumping effect» is highly likely. The situation becomes completely different if the functional dependence $\theta(\psi)$ proposed by Haverkamp et al. (1977) is used, or its

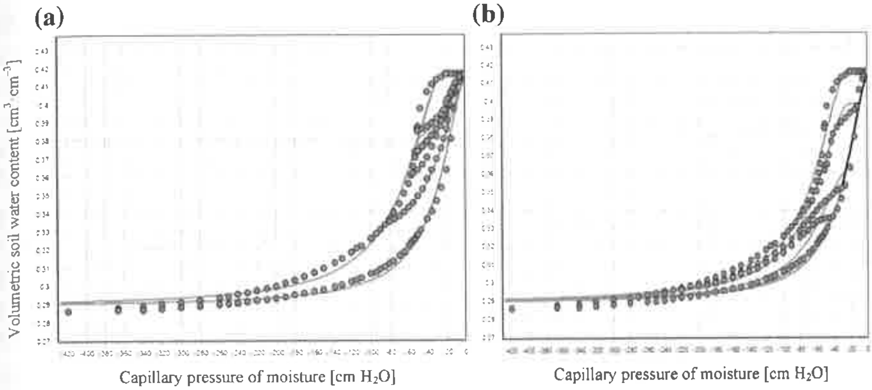

Fig. 17.3 **a** Approximation (dot-fitting) of data about the boundary branches; predictive estimation of the four wetting scanning branches for soil *Rideau clayey loam* using the proposed model. **b** Approximation (dot-fitting) of data about the boundary branches; predictive estimation of the three drying scanning branches for soil *Rideau clayey loam* using the proposed model

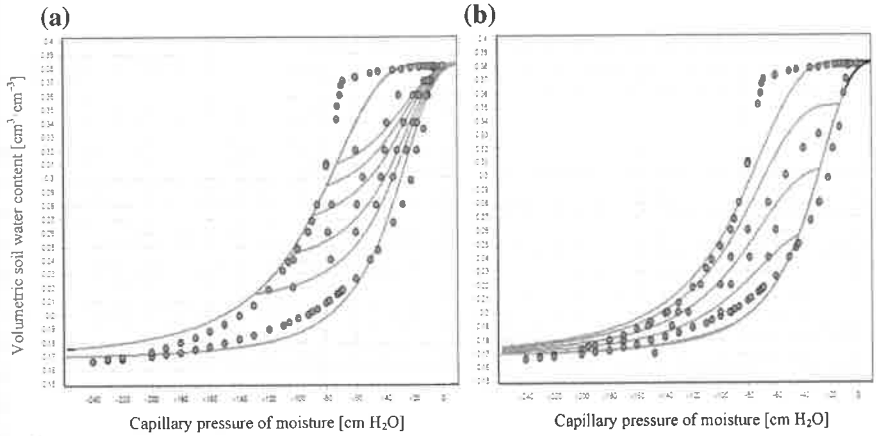

Fig. 17.4 **a** Approximation (dot-fitting) of data about the boundary branches; predictive estimation of the five wetting scanning branches for soil *Rubicon sandy loam* using the proposed model. **b** Approximation (dot-fitting) of data about the boundary branches; predictive estimation of the four drying scanning branches for *soil Rubicon sandy loam* using the proposed model

modified version described by formulas (17.1) and (17.2). In this case, calculating the hysteresis loop of the soil water-retention capacity using these dependencies reduces to the problem of an adequate estimation of the values of the parameters of these dependence. And if a solution is found for this problem, then the «pumping effect» does not appear.

The authors of this study are very sceptical of the idea that the hysteresis loops formed by the scanning branches should be artificially closed (Huang et al. 2005). It is clear that in this case there is no «pumping effect». But then we cannot expect the physical adequacy of the model. Indeed, if the different scanning (secondary) drying branches «come» to the same point, for example, on the main drying branch, then the meaning of the function of the differential moisture capacity of the soil disappears, because it has infinitely many values. According to the authors of this study, not a single scanning branch can cross the main branches of the hysteresis loop of the soil water-retention capacity. But the intersection between the scanning drying and wetting branches is possible. Oscillation of the values of the capillary pressure of soil moisture between the points of neighbouring intersections can lead to a certain «drift» of such a loop, which gradually decreases, asymptotically approaching an infinitesimal value. That is, with such oscillations, the closed loop should gradually localize in a certain area of the hysteresis loop, but not beyond the physically acceptable boundaries. Moreover, at each point on any branch of hysteresis, the function of the differential moisture capacity of the soil must take only two values: one for drying, and the other for wetting the soil (using two sets of physically interpreted parameters) (Terleev et al. 2016c, 2017a, b, 2018a, b). These ideas form the basis of the model proposed by the authors of this study.

17.5 Conclusion and Outlook

The mathematical model proposed by the authors of this paper corresponds to physical concepts in relation to the phenomenon of hysteresis of the soil water-retention capacity. Undesirable artificial «pumping effect» is not revealed. The application of irrigation precise rates calculated with the help of this model prevents the percolating the excess moisture under the action of gravity beyond the rooting zone of the soil, which minimizes the unproductive losses of irrigation water. The present model can be integrated into existing irrigation scheduling programs and systems such as BEREST90 or WEB-BEREST (Wenkel and Mirschel 1991; Mirschel et al. 2014a) used in German agriculture. It is also important for the estimation of irrigation water demand and additional crop yields due to irrigation under climate change (Mirschel et al. 2014b). The use of the proposed model in the development of farming technologies, as well as in substantiating land reclamation measures, will contribute to optimizing the water-air and nutrient regimes of the soil, as well as the rational use of water resources and agrochemicals.

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References

Ahuja LR, Swartzendruber D (1972) An improved form of soil-water diffusivity function. Soil Sci Soc Am Proc 36:9–14

Brutsaert W (1966) Probability laws for pore-size distribution. Soil Sci 101:85–92

D'Hollander EH (1979) Estimation of the pore size distribution from the moisture characteristic. Water Resour Res 15:107–112

Gillham RW, Klute A, Heermann DF (1976) Hydraulic properties of a porous medium: measurement and empirical presentation. Soil Sci Soc Am J 40:203–207

Haverkamp R, Vauclin M, Touma J, Wierenga PJ, Vachaud G (1977) A comparison of numerical simulation model for one-dimensional infiltration. Soil Sci Soc Am J 41:285–294

Huang HC, Tan YC, Chen CH (2005) A novel hysteresis model in unsaturated soil. Hydrol Process 19:1653–1665

Kool JB, Parker JC (1987) Development and evaluation of closed-form expressions for hysteretic soil hydraulic properties. Water Resour Res 23:105–114

Kosugi K (1994) Three-parameter lognormal distribution model for soil water retention. Water Resour Res 30:891–901

Kosugi K (1996) Lognormal distribution model for unsaturated soil hydraulic properties. Water Resour Res 32:2697–2703

Kosugi K, Hopmans JW (1998) Scaling water retention curves for soils with lognormal pore-size distribution. Soil Sci Soc Am J 62:1496–1505

Mirschel W, Klauss H, Berg M, Eisenhut K-U, Issbrücker G, Prochnow A, Schörling B, Wenkel K-O (2014a) Innovative Technologien für eine effiziente Bewässerung im Pflanzenbau. In: Bloch R, Bachinger J, Fohrmann R, Pfriem R (eds) Land- und Ernährungswirtschaft im Klimawandel—Auswirkungen, Anpassungsstrategien und Entscheidungshilfen. oekom verlag, München, pp 261–277

Mirschel W, Wieland R, Wenkel K-O, Nendel C, Guddat C (2014b) YIELDSTAT—a spatial yield model for agricultural crops. Eur J Agron 52:33–46

Mualem Y (1976) A catalogue of the hydraulic properties of unsaturated soils. Research Project 442. Technion, Israel Institute of Technology, Haifa, Israel, p 100

Scott PS, Farquhar GJ and Kouwen N (1983) Hysteretic effects on net infiltration. In: Proceeding of national conference on advances in infiltration publication 11-83 (Michigan: St. Joseph American Society of Agricultural Engineers), pp 163–170

Terleev VV, Topaj AG, Mirschel W (2015) The improved estimation for the effective supply of productive moisture considering the hysteresis of soil water-retention capacity. Russ Meteorol Hydrol 40:278–285

Terleev V, Petrovskaia E, Nikonorov A, Badenko V, Volkova Y, Pavlov S, Semenova N, Moiseev K, Topaj A, Mirschel W (2016a) Mathematical modeling the hydrological properties of soil for practical use in the land ecological management. In: MATEC web of conferences 73, Article number 03001

Terleev V, Nikonorov A, Togo I, Volkova Y, Garmanov V, Shishov D, Pavlova V, Semenova N, Mirschel W (2016b) Modelling the hysteretic water retention capacity of soil for reclamation research as a part of underground development. Procedia Eng 165:1776–1783

Terleev V, Petrovskaia E, Sokolova N, Dashkina A, Guseva I, Badenko V, Volkova Yu, Skvortsova O, Nikonova O, Pavlov S, Nikonorov A, Garmanov V, Mirschel W (2016c) Mathematical modeling of hydrophysical properties of soils in engineering and reclamation surveys. In: MATEC web of conferences 53, Article no 01013

Terleev VV, Nikonorov AO, Togo I, Volkova YuV, Ginevsky RS, Lazarev VA, Khamzin ER, Garma-nov VV, Mirschel W, Akimov LI (2017a) Hysteretic water-retention capacity of sandy soil. Mag Civ Eng 2:84–92

Terleev V, Ginevsky R, Lazarev V, Nikonorov A, Togo I, Topaj A, Moiseev K, Abakumov E, Melnichuk A, Dunaieva I (2017b) Predicting the scanning branches of hysteretic soil water-retention capacity with use of the method of mathematical modeling. In: IOP conference series: earth and environmental science 90, Article no 012105

Terleev VV, Nikonorov AO, Ginevsky RS, Lazarev VA, Togo I, Topaj AG, Moiseev KG, Pavlova VA, Layshev KA, Arkhipov MV, Yu Melnichuk A, Dunaieva IA, Mirschel W (2018a) Hysteresis of the soil water-retention capacity: estimating the scanning branches. Mag Civ Eng 1:141–148

Terleev V, Mirschel W, Nikonorov A, Ginevsky R, Lazarev V, Topaj A, Moiseev K, Layshev K, Arkhipov M, Melnichuk A, Dunaieva I, Popovych V (2018b) Five models of hysteretic water-retention capacity and their comparison for sandy soil. In: MATEC web of conferences 193, Article no 02036

Van Genuchten M Th (1980) A closed form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Sci Soc Am J 44:892–989

Wenkel K-O, Mirschel W (1991) BERSIM und BEREST 90 Modelle zur Simulation der Bodenfeuchte und Evapotranspiration sowie zur operativen Beregnungseinsatzsteuerung im integrierten Landbau. Agrarinformatik 21:349–360

Winitzki S (2008) A handy approximation for the error function and its inverse (in https://sites.google.com/si-te/winitzki/sergei-winitzkis-files/erf-approx.pdf?attredirects=0)